RECONSTRUCTION CONJECTURE

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RECONSTRUCTION CONJECTURE

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CERTIFICATE

This is to certify that the M.Phil. thesis entitled 'Reconstruction Conjecture' by Mr. Amar Singh Sisodia is a record of bonafide research work carried out by him under my supervision and guidance. He had fulfilled the other requirements for the award of M.Phil. degree. The results embodied in this thesis have not been submitted to any other Institute or University for the award of any degree or diploma.

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Amar Singh Sisodia.

Amar Singh Sisodia

CONTENTS

Chapter 1

INTRODUCTION

In this chapter we introduce the problem, give some elementary definitions and discuss about the point-reconstruction and reconstruction of valencies.

1.1 Preliminary remarks

The Reconstruction Conjecture is generally regarded as one of the foremost unsolved problems in Graph-Theory. Indeed, Harary in his book 'Proof Techniques in Graph-Theory' (54) has even classified it as a "Graphical disease" because of its contagions nature.

According to reliable sources, it was discovered in Wisconsion in 1941 by Kelly and Ulam, and claimed its first victum P.J.Kelly in 1942 (67).

Our purpose here is to discuss the current status of the Conjecture. We will also obtain some new results and suggest some unsolved problems. We shall, for the most part, use the terminology and notations of Harary (53).

To date, all graphs with at most nine points have been checked (see,(68),(60),(95) and (104)) and no counter

example has been found. Furthermore, Muller (97) has shown that almost all graphs are reconstructible.

In this chapter, we give some elementary definitions and then discuss the point reconstruction in view of Kelly's lemma and Counting theorem. Then some important results of Nash-Williams about reconstruction and valencies are given.

The Edge-reconstruction Conjecture and reconstruction of digraphs is discussed in chapter 2.Which also includes Muller's result that the edge-reconstruction conjecture is in a certain sense true for almost all graphs. For the digraph, Stockmeyer's results show the Conjecture to be false for some arbitrarily large digraph. Some more results of Muller are also there.

In chapter 3, we obtain some new results. They include reconstruction of Complete point graphs, Euler graphs, Odd graphs and Non-consecutive graphs.

In chapter 4, we discuss the reconstruction of Complete bipartite graphs and then suggest some unsolved problems related to the reconstruction Conjecture.

At the end we give a complete up to date bibliography on the problem.

1.2 Elementary definitions

Definition (1.2.1)

A graph G consists of a finite non-empty set $V = V(G)$ of points together with a prescribed set X of unordered pairs of distinct points of V. Each pair $e = \{u,v\}$ of points in X is a line of G. We write $e = uv$ and say that u and v are adjacent points. And e is incident with u and v.

Notations

We frequently use the words vertex and edge for the point and line respectively. The point set of the graph G is denoted by V(G) and the line set by E(G). We denote $|V(G)| = \nu$ and $|E(G)| = \epsilon$. A graph with p points and q lines is called a(p,q) graph. Unless otherwise stated G is always a(p,q) graph in our discussions.

Definition (1.2.2)

A subgraph of G is a graph having all of its points and lines in G.

Definition (1.2.3)

A spanning subgraph is a subgraph containing all the points of G.

Definition (1.2.4)

Two graphs G and H are isomorphic if there exists a one-to-one correspondence between their point sets which preserves adjacency. We denote it by $G \cong H$.

Definition (1.2.5)

The degree of a point v_i in graph G, denoted d_i or deg v_i or $d(v_i)$, is the number of lines incident with v_i. Sometimes we also use the word valency for the degree.

Theorem (1.2.1) (Euler) (53)

The sum of the degrees of the points of a graph G is twice the number of lines, that is, $\sum_{i=1}^{p} \deg v_i = 2q$ .... (1.2.1)

Proof

Since every line is incident with two points, it contributes 2 to the sum of the degrees of the points //.

Corollary (1.2.1.1) (53)

In any graph, the number of points of odd degree is even.

Definition (1.2.6)

The point $v \in G$ is isolated if deg $v = 0$.

Definition (1.2.7)

The point $v \in G$ is end point if deg $v = 1$.

Definition (1.2.8)

If all points of the graph G have the same degree, then G is called a regular graph.

Definition (1.2.9)

A regular graph in which all the points have degree 3 is called a cubic graph.

Definition (1.2.10)

The complete graph has every pair of its points adjacent. It is denoted by K_p. Thus K_p has $\binom{p}{2}$ lines and is a regular graph of degree p-1.

Definition (1.2.11)

The complement $\bar{G}$ of a graph G also have V(G) as its point set, but two points are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

Definition (1.2.12)

A path of a graph G is an alternating sequence of distinct points and lines $v_o, x_1, v_1, \ldots, v_{n-1}, x_n, v_n$, beginning and ending with points, in which each line is incident with the two points immediately preceding and following it. This path joins v_o and v_n, and may also be denoted by $v_o v_1 v_2 \ldots . v_n$; it is sometimes called a $v_o - v_n$ path.

Definition (1.2.13)

If in a path $v_o = v_n$ and $n \geqslant 3$ then it is a cycle. If G is a cycle then all its points are of degree 2.

Definition (1.2.14)

A graph is connected if every pair of points are joined by a path ; otherwise it is disconnected.

Definition (1.2.15)

A maximal connected subgraph of G is called a connected component or simply a component of G.

Definition (1.2.16)

A graph G is said to be a disconnected graph if it has at least two components.

Definition (1.2.17)

A graph is acyclic if it has no cycles.

Definition (1.2.18)

A tree is a connected acyclic graph.

Definition (1.2.19)

A graph G which is connected and has exactly one cycle is called a unicyclic graph.

Definition (1.2.20)

A cut point of a graph is one whose removal increases the number of components.

Definitions (1.2.21)

A non-separable graph is connected, nontrivial, and has no cut points.

Definition (1.2.22)

The connectivity $k = k(G)$ of a graph G is the minimum

number of points whose removal results in a disconnected or trivial graph.

Definition (1.2.23)

A graph G is n-connected if $k(G) \geq n$.

1.3 Point-reconstruction

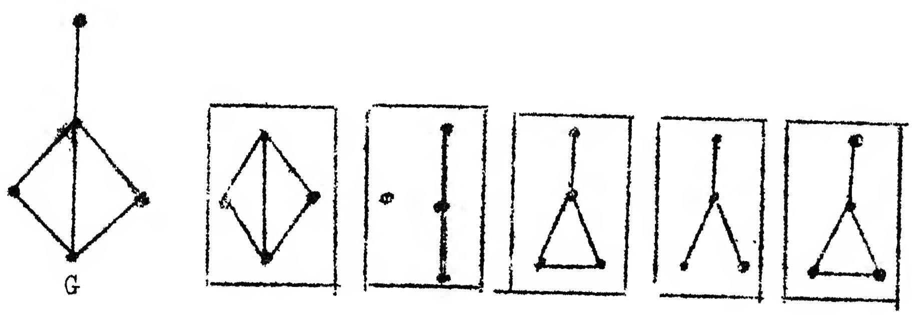
Definition (1.3.1)

An ordinary graph is a graph with at least three points.

Definition (1.3.2)

A subgraph of G obtained by deleting a point v together with its incident lines will be referred to as a point deleted subgraph and denoted by G_v (rather than $G-v$).

Figure (1.3.1) exhibits the point-deleted subgraphs of a graph G.



The point-deleted subgraphs of the graph G

Figure (1.3.1)

The Reconstruction Conjecture (First Version)(1.3.1)

If G and H are two ordinary graphs, and if there exists a bijection $r : V(G) \rightarrow V(H)$ such that $G_v \cong H_{r(v)}$ for every $v \in V(G)$, then $G \cong H$.

Definition (1.3.3)

A reconstruction of a graph G is a graph H such that $V(H) = V(G)$ and $H_v \cong G_v$ for all $v \in V(G)$.

Definition (1.3.4)

A graph G is said to be reconstructible if every reconstruction of G is isomorphic to G.

The Reconstruction Conjecture (Second Version)(1.3.2)

Every ordinary graph is reconstructible.

It is helpful to imagine a deck of cards on which the point-deleted subgraphs of a graph G are drawn, but not labelled. Presented with such a deck, first we have to find some graph which produces that deck and then to show that, regardless of the algorithm used, one necessarily ends up with the same graph.

Definition (1.3.5)

A hypomorphism of a graph G onto a graph H is a bijection $r : V(G) \rightarrow V(H)$ such that $G_v \cong H_{r(v)}$ for every $v \in V(G)$.

Definition (1.3.6)

Two graphs G and H are hypomorphic if there exists a hypomorphism of G onto H.

Definition (1.3.7)

A graph G is reconstructible if every graph hypomorphic to G is isomorphic to G.

Throughout our discussions, we imagine a graph G with p points $v_1, v_2, \ldots, v_p$ and q lines, giving rise to a deck of p cards $C_1, C_2, \ldots, C_p$ such that a graph isomorphic to G_{v_i} is drawn on the card C_i for each $i = 1, 2, \ldots p$ we will call it G_i for simplicity. Let the card C_i have q_i lines for each $i = 1, 2, \ldots, p$.

Theorem (1.3.1)

Let G be an ordinary (p,q) graph and G_i's for $i = 1, 2, \ldots, p$ be its point-deleted subgraphs. Let G_i have q_i lines for each $i = 1, 2, \ldots, p$, then

$$q = \frac{\sum_{i=1}^{p} q_i}{(p-2)} \qquad \ldots\ldots(1.3.1)$$

Proof

Let G have points $v_1, v_2, \ldots, v_p$ with valencies $d_1, d_2, \ldots, d_p$, then $q_i = q - d_i$ for all $i = 1, 2, \ldots, p$.

Taking summation over i from 1 to p on both the sides,

$$\sum_{i=1}^{p} q_i = \sum_{i=1}^{p} (q - d_i) = pq - \sum_{i=1}^{p} d_i = pq - 2q$$

(by equation (1.2.1))

Hence $$q = \frac{\sum_{i=1}^{p} q_i}{(p-2)} \quad //.$$

Theorem (1.3.2) (99)

(i) If G and H are hypomorphic graphs, then

$|V(G)| = |V(H)|$ = Number of cards in the deck.

(ii) If G and H are hypomorphic graphs, then they are both connected or both disconnected.

Definition (1.3.8)

We call a class R of graphs reconstructible if each graph in R is reconstructible.

Definition (1.3.9)

A parametter, or indeed any function defined on a class R of graphs, is reconstructible if, for each graph G in R, it takes the same value on all reconstructions of G.

Kelly's Lemma (1.3.1) (14)

For any two graphs F and G such that $|V(F)| < |V(G)|$, the number $S(F,G)$ of subgraphs of G isomorphic to F is reconstructible.

Corollary (1.3.1.1) (14)

For any two graphs F and G such that $|V(F)| < |V(G)|$, the number of subgraphs of G which are isomorphic to F, and include a given point v, is reconstructible.

Definition (1.3.10)

A class R of graphs is recognizable if, for each graph G in R, every reconstruction of G is also in R.

number $m(F,G)$ of maximal T-subgraphs of G isomorphic to F is reconstructible.

Corollary (1.3.2.1) (14)

Disconnected graphs are reconstructible.

Theorem (1.3.3) (Kelley) (68)

Trees are reconstructible.

Theorem (1.3.4) (Manvel)(84)

Unicyclic graphs are reconstructible.

Greenwell and Hemminger (49) generalize the result of Manvel (84). Further extensions, obtained by refining the methods of Greenwell and Hemminger have been given by Krishnamoorthy (71) and Krishnamoorthy and Parthasarathy (73).

Theorem (1.3.5) (Greenwell and Hemminger)(50)

If F is n-connected but G is not, the number of maximal n-connected subgraphs of G isomorphic to F is reconstructible.

Definition (1.3.16)

For a separable graph G which is not a tree, the trunk of G be the maximal subgraph of G that has no points of degree one.

Definition (1.3.17)

A limb of G is a maximal subtree that contains exactly one point of the trunk; this point is the root of the limb.

Theorem (1.3.6) (Bondy) (11)

The trunk and the rooted limbs are reconstructible.

Bondy (11) also proved that separable graphs are reconstructible in certain special cases.

1.4 Reconstruction and Valencies

Definition (1.4.1)

The valency-sequence of a graph is the sequence obtained by listing the valencies of its points in non decreasing order.

The information on the cards can be used to determine the valencies of the points of G. To do this, first determine q from the information on the cards, as explaned in Theorem (1.3.1). From q subtract the number of lines on any card C_i to discover the valency d_i of the corresponding point v_i, that is, $d_i = q-q_i$ for all $i=1,2,..p$.

Theorem (1.4.1) (99)

If r is a hypomorphism of an ordinary graph G onto an ordinary graph H, then $d(v) = d(r(v))$ for every $v \in V(G)$.

Corollary (1.4.1.1) (99)

Hypomorphic ordinary graphs have the same valency-sequence.

Definition (1.4.2)

The neighbourhood of a point v in G, denoted by $N_G(v)$, is the set of points of G adjacent to v. The points of $N_G(v)$ are called neighbours of point v.

Definition (1.4.3)

The valency-sequence of a point v (denoted by $v_{S(v)}$ or $v_{S_G(v)}$) of a graph G be the sequence obtained by listing the valencies of the neighbours of v in non-decreasing order.

Definition (1.4.4)

The valency-sequence sequence of a graph G is defined to be the sequence of sequences obtained by listing the valency-sequences of the points of G in dictionary order.

The information on the cards suffices to determine the valency-sequences of points. For, having ascertained the valencies of $v_1, v_2, \ldots, v_p$, we can determine, for any non-negative integer k, the number of k-valent neighbours of a point v_i. To do so, count those points which have valency less than k in the graph depicted on G_i, and subtract the number of points other than v_i whose valency in G is less than k (the valency of v_i itself being irrelevent at this point). By doing this for k = o, 1,2,...,p-1, we discover the valency-sequence of v_i.

Lemma (1.4.1)(99)

If G and H are hypomorphic ordinary graphs, $v \in V(G)$, $w \in V(H)$, and $G_v \cong H_w$, then $v_{S_G(v)} = v_{S_H(w)}$.

Theorem (1.4.2) (99)

If r is a hypomorphism of an ordinary graph G onto an ordinary graph H, then $v_{S_G(v)} = v_{S_H(r(v))}$, for every $v \in V(G)$.

Corollary (1.4.2.1)(99)

Hypomorphic ordinary graphs have the same valency-sequence sequence.

Definition (1.4.5)

Let v be a point of agraph G. Then a v-reconstruction of G is a graph H such that $V(G) = V(H)$, $G_v \cong H_v$, and G is hypormorphic to H.

Lemma (1.4.2)(99)

A graph G is reconstructible if it has a point v such that every v-reconstruction of G is ismorphic to G.

Definition (1.4.6)

A point v_i of a graph G is bad or good according as G does, or does not, possess a point of degree d_i-1.

Lemma (1.4.3) (99)

Let v be a point of an ordinary graph G, and let H be a v-reconstruction of G. Then, for some non-negative integer r, there exist r distinct bad neighbours $v_1, v_2, \ldots, v_r$ of v in G, and r distinct points $w_1, \ldots, w_r$ of $V(G)-N_G(v)$ such that $d(w_i) = d(v_i)-1$ for $i = 1,2,\ldots,r$ and $N_H(v) = (N_G(v) - \{v_1, \ldots v_r\}) \cup \{w_1, \ldots, w_r\}$.

Theorem (1.4.3)(99)

An ordinary graph is reconstructible if it has a point all of whose neighbours are good.

In the next chapter we will discuss about the Edge-reconstruction and reconstruction of digraphs.

Chapter 2

EDGE AND DIGRAPH RECONSTRUCTION

In this chapter we discuss the edge-reconstruction of the graphs and the reconstruction of digraphs.

2.1 Edge-reconstruction

A related, and seemingly easier, Conjecture was put forward by Harary 1964 (51). Here one is presented with a deck of cards bearing, not the point deleted subgraphs of a given graph, but its edge-deleted subgraphs. As before, one is asked to determine the graph from this information.

Definition (2.1.1)

If e is an edge of a graph G, then the graph $G-(e)$ obtained by deleting e from G will be called an edge-deleted subgraph of G. Thus $V(G-e) = V(G)$ and $E(G-e) = E(G)-(e)$. For simplicity, we denote $G-(e)$ by G_e.

The Edge-reconstruction Conjecture (2.1.1)

If G and H are graphs with at least four edges, and if there exists a bijection $r : E(G) \to E(H)$ such that $G_e \cong H_{r(e)}$ for every $e \in E(G)$, then $G \cong H$.

If graphs with fewer than four edges were allowed, the graphs G and H of Figure (2.1.1) would constitute a counter example to this Conjecture, and so would the graph G and H of Figure (2.1.2).

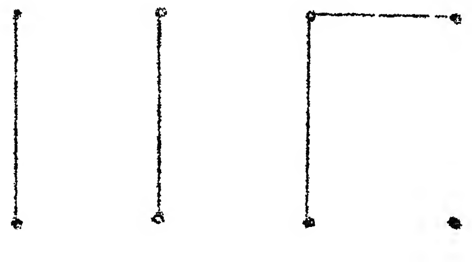


Figure (2.1.1)

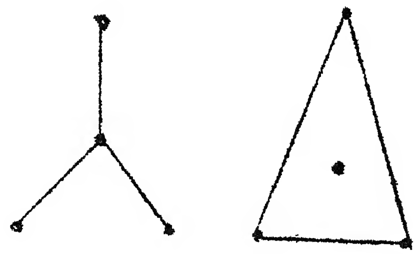


Figure (2.1.2)

Harary's formulation of the Conjecture (2.1.2)

All simple finite graphs with at least four edges are edge reconstructible.

In the remainder of this section we confine our attention to finite simple undirected graphs with at least four edges.

Definition (2.1.2)

Let L(G) be the line graph of the graph G, then the points of L(G) are the lines of G, with two points of L(G) adjacent whenever the corresponding lines of G are.

Theorem (2.1.1) (64)

A graph is edge reconstructible if and only if its edge graph(that is line graph) is reconstructible.

Kelly's Lemma (for edges) (2.1.1)

For any two graphs F and G such that $\mathcal{E}(F) < \mathcal{E}(G)$, the number of subgraphs of G isomorphic to F is edge reconstructible.

Definition (2.13)

A subgraph F of a graph G is said to be edge proper if $\mathcal{E}(F) < \mathcal{E}(G)$.

Counting Theorem (for edges) (2.12) (14)

Let R be an edge-recognizable class of graphs, and let T be any class of graphs such that, for every G in R, each T-subgraph of G is (i) edge proper (ii) contained in a unique maximal T-subgraph of G. Then, for every F in T and every G in R, the number of maximal T-subgraphs of G isomorphic to F is edge reconstructible.

The number of isolated points is readily shown to be edge reconstructible. By Kelly's lemma, one can determine whether or not the longest path in G has length one, two or more. The number of isolated points of G is accordingly m-2, m-1, or m, where m denotes the minimum number of isolated points in any G_e.

Theorem (2.13) (14)

Edge reconstruction Conjecture is valid for all graphs if it is valid for graphs without isolated points.

Theorem (2.1.4) (48)

If G is reconstructible and has no isolated points, then G is edge reconstructible .

With the help of this theorem, we deduce that several classes of graphs are edge reconstructible ; regular graphs, disconnected graphs with at least two non-trivial components, trees, certain products (see also (32)).

Theorem (2.1.5) (Muller) (98)

All graphs, except possibly those with relatively few edges, are edge reconstructible.

Proof is due to Muller (98) based on an ingenious application of the inclusion-exclusion principle by Lovász (82). We can also derive it from a theorem of Nash-Williams (99) //.

Definition (2.1.4)

If G and H are graphs and F is a spanning subgraph of G, we let $(G \to H)_F$ denote the set of injections $r : V(G) \to V(H)$ such that, for each edge uv of G, r(u)r(v) is an edge of H if and only if uv is an edge of F.

Definition (2.1.5)

For convenience, we denote the set of embeddings of G into H (that is, $(G \to H)_G$) by $G \to H$, and the number $|(G \to H)_F|$ by $|G \to H|_F$.

Definition (2.1.6)

We shall also write $F \subseteq G$ to indicate that F is a spanning subgraph of G.

Theorem (Nash-Williams) (2.1.6) (96)

G is edge reconstructible if either of the following two conditions holds :

(i) There exists a spanning subgraph F of G such that $|G \to H|_F = |G \to G|_F$ for every edge reconstruction H of G ;

(ii) There exists a spanning subgraph F of G such that $\varepsilon(G) - \varepsilon(F)$ is even and $|G \to G|_F = 0$.

Corollary (Lovasz) (2.1.6.1) (82)

G is edge reconstructible if $\varepsilon > (1/2)\binom{\nu}{2}$.

Schmeichel (114) has strengthened corollary (2.1.6.1) a little by giving a condition on the degree sequence of G which forces $|G \to H^c|$ to be zero for every edge reconstruction H of G (namely, $d_i + d_{v+1-i} \geq \nu$ for some i, where $d_1 \leq d_2 \leq \ldots \leq d_\nu$).

Corollary (Muller) (2.1.6.2) (98)

G is edge reconstructible if $2^{\varepsilon-1} > \nu!$.

Corollary (2.1.6.3) (99)

G is edge reconstructible if

$$\varepsilon > (\nu \log \nu)/(\log 2).$$

2.2 Digraph reconstruction

Definition (2.2.1)

A digraph D consists of a finite set V of points and a collection of ordered pairs of distinct points of V. Any pair (u,v) is called a directed line and denoted by uv.

Definition (2.2.2)

The out-valency od(v) of a point v is the number of points adjacent from it, and the in-valency id(v) is the number adjacent to it.

Definition (2.2.3)

A digraph is strongly connected if every two points are mutually reachable. It is unilateral, if for any two points at least one is reachable from the other, and is weakly connected if every two points are joined by a semipath. It is disconnected if it is not even weakly connected.

Definition (2.2.4)

A tournament is an oriented complete graph.

Definition (2.2.5)

Two digraphs D and D' constitute a counter example to the reconstructibility of digraphs if $D \not\cong D'$ but there exists a bijection $r : (V(D) \to V(D')$ such that $D_v \cong D'_{r(v)}$ for every $v \in V(D)$.

The question of reconstruction applies equally well to directed graphs as to undirected graphs. Here in addition to several other small counter example pairs, one finds pairs of tournaments on three and four points. (See Figure (2.2.1) and Figure (2.2.2)). Note that both Kelly's lemma and the Counting theorem apply (with appropriate modifications) to digraphs. In the Counting theorem we shall denote the classes of digraphs by D and T. In view of these counter examples, it is natural to consider first the reconstruction of tournaments.

<u>Theorem (Harary and Palmer) (2.2.1) (62)</u>

A tournament with at least five points is reconstructible if it is not strongly connected.

<u>Conjecture (Harary and Palmer)(2.2.1)(62)</u>

A strongly connected tournament with at least five points is reconstructible.

This conjecture was disproved by Beineke and Parker (4) when they discovered counter example pairs on five and six points. See Figure (2.2.3) and Figure (2.2.4). Then Harary (55) renewed the Conjecture (2.2.1) by excluding these counter examples. It was then challenged by Stockmeyer (125) who found two pairs on eight points by means of an exhaustive computer search (which also determined that all tournaments on seven points are reconstructible). Finally, Stockmeyer (126),(127) administered the Coup de grace by constructing non reconstructible

tournaments on 2^n+2^m points, for all m and n not both zero. Stockmeyer (127) also notes that there are many non-reconstructible digraphs other than tournaments.

Definition (2.2.6)

We define the valency-pair of a point v of a digraph D to be the ordered pair $(od(v), id(v))$.

Definition (2.2.7)

We define the valency-pair sequence of a digraph D to be the sequence of $|V(D)|$ ordered pairs of numbers obtained by listing the valency pairs of the points of D in dictionary order.

Theorem (Manvel) (2.2.2) (88)

If D,D' are digraphs with at least five points, and if there exists a bijection $r : V(D) \to V(D')$ such that $D_v \cong D'_{r(v)}$ for each $v \in V(D)$, then D and D' have the same Valency pair sequence.

On the positive side Manvel (88),(90),(91) proved Theorem (2.2.2) and that the connectedness type (strongly connected, unilateral, weakly connected or disconnected) is recognizable.

In the next chapter we prove some classes of graphs to be reconstructible. They include Complete point graphs, Euler graphs, Odd graphs and Non-consecutive graphs.

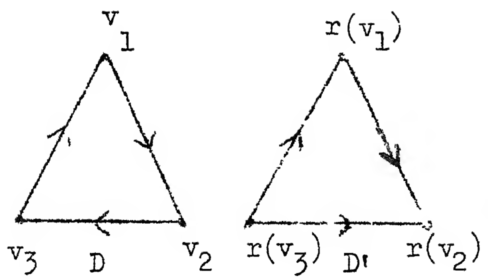


Figure (2.2.1)

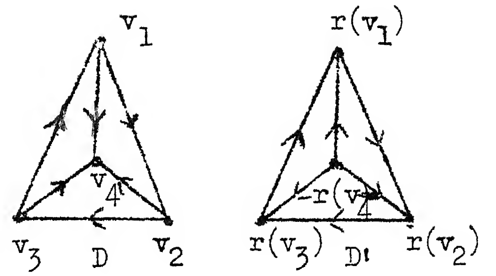


Figure (2.2.2)

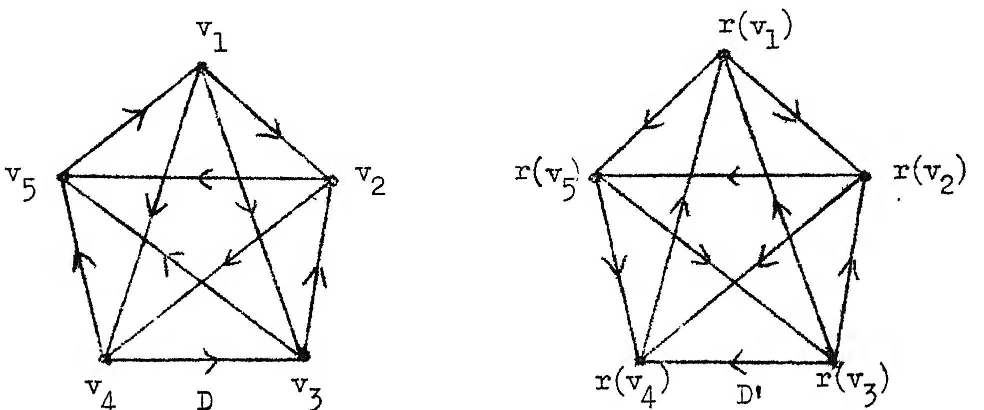


Figure (2.2.3)

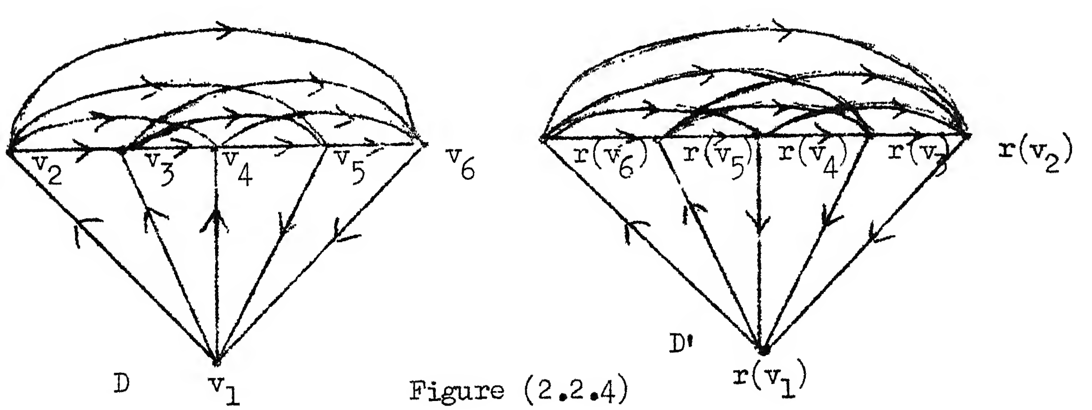


Figure (2.2.4)

Nonreconstructible tournaments with three, four, five and six vertices.

Chapter 3

SOME CLASSES OF RECONSTRUCTIBLE GRAPHS

In this chapter we define Complete point graphs, Odd graphs and Non-consecutive graphs. We prove that these graphs and Euler graphs are reconstructible.

This is to increase the class of known reconstructible graphs, and possibly also to suggest the search for a counter example to the reconstruction Conjecture if one thinks it to be false. We must however confess that, in some tentative explorations in the direction we have hitherto obtained only a few results.

3.1 Complete point graphs

Definition (3.1.1)

Let G be a graph. A point $v \in G$ is said to be complete if it is adjacent with all the other points of the graph G. That is, if G is a (p,q) graph, then $v \in G$ is complete if and only if $d(v) = p-1$.

Definition (3.1.2)

A graph G is said to be a Complete point graph if there exists a complete point $v \in G$.

Definition (3.1.3)

A graph G is said to be Complete if and only if all of its points are complete.

Lemma (3.1.1) given below shows that Complete point graphs are recognizable.

Lemma (3.1.1)

G is a Complete point graph if and only if at least one G_i has $q_i = q-(p-1)$.

Proof

Let G have a complete point v_i then G_i will have $q_i = q-d(v) = q-(p-1)$.

Conversely, let at least one G_i have $q_i = q-(p-1)$, then in the degree sequence of G we have,

$$d_i = q-q_i = q- \{q-(p-1)\} = (p-1).$$

Hence at least one point in G has degree $(p-1)$. Therefore, G is a Complete point graph //.

Theorem (3.1.1)

Complete point graphs are reconstructible.

Proof

Let G be a Complete point graph. Then by Lemma (3.1.1), there exists at least one G_i such that $q_i = q-(p-1)$. Choose any card for which $q_i = q-(p-1)$, that is, a card corresponding

to a complete point. Then, in order to reconstruct G from this card insert the point v_i. In order that v_i should have degree $q-q_i = (p-1)$, v_i is to be adjacent with $(p-1)$ points of G_i. But there is a unique way to join v_i with respect to this condition. Join v_i to all the points of the subgraph G_i. As the method of reconstruction is unique, all the possible reconstructions are isomorphic //.

Corollary (3.1.1.1)

Complete graphs are reconstructible.

Proof

Every Complete graph is a Complete point graph//.

3.2 Euler graphs and Odd graphs

Here we will consider only connected graphs because disconnected graphs are shown to be reconstructible in Corollary (1.3.2.1).

Definition (3.2.1)

A connected graph G is Euler or Even if all of its points are of even degree.

Definition (3.2.2)

A connected graph G is Odd if all of its points are of odd degree.

Lemma (3.2.1) given below shows that, Euler (Odd) graphs are recognizable.

Lemma (3.2.1)

G be Euler (Odd) if and only if its degree sequence reconstructed from G_i's have only even (odd) integers in it.

Proof

The proof is obvious from the definitions. We can find the degree sequence of G with the help of all G_i's and hence recognize that whether G is Euler (Odd) or not //.

Theorem (3.2.1)

Euler (Odd) graphs are reconstructible.

Proof

Let G be an Euler (Odd) graph then by Lemma (3.2.1) its degree sequence reconstructed from G_i's has only even (odd) integers in it. Choose any one card from the deck. Let the card be G_i with having q_i lines in it. Then $d_i = q-q_i$.

Now we notice that from G when this point v_i was deleted, then exactly d_i points lose there degrees by one and $(p-1)-d_i$ points remain unchanged. Therefore on the card G_i we can see that $(p-1)-d_i$ points are of even (odd) degrees and exactly d_i points are of odd (even) degrees.

Now to reconstruct G from G_i insert the point v_i. In order to get G as an Euler (Odd) graph join v_i to all the points of odd (even) degrees. The points of odd (even) degree are d_i in number. In the case of Euler graphs therefore d_i

must be even as it is the number of odd points in G_i.

In the case of Odd graphs d_i must be odd because Odd graphs always have even number of points. Therefore p is even. And in G_i we have that $(p-1)-d_i$ points are of odd degrees. Therefore $(p-1)-d_i$ must be even. Now p is even, $(p-1)-d_i$ is even, then $p-\{(p-1)-d_i\}-1 = d_i$ must be odd.

Thus in both the cases we get our original graph G which will be Euler (Odd).

The reconstruction is unique as there is a unique way to join the point v_i with the d_i points which are having odd (even) degrees. Hence all the possible reconstructions are isomorphic //.

Some interesting corollaries of the theorem (3.2.1) can be given like the following.

<u>Corollary (3.2.1.1)</u>

Complete graphs are reconstructible.

<u>Proof</u>

Every Complete graph will be either an Euler or an Odd graph. Hence it is reconstructible //.

<u>Corollary (3.2.1.2)</u>

Cycles are reconstructible.

<u>Proof</u>

Every cycle is an Euler graph, all of whose points have degree 2. Hence it is reconstructible //.

Corollary (3.2.1.3)

Cubic graphs are reconstructible.

Proof

Every cubic graph is an Odd graph, all of whose points have degree 3. Hence the result //.

Corollary (3.2.1.4)

Regular graphs are reconstructible.

Proof

Every regular graph will be either an Euler or an Odd graph. Hence reconstructible //.

Corollary (3.2.1.5)

A graph G in which the subgraphs G_i's in all the cards of the deck are isomorphic, is reconstructible.

Proof

We see in this case q_i, the number of lines in each G_i, is the same and therefore $d_i = q-q_i$ is also the same for each point v_i, and the graph will be a regular graph. Hence by Corollary (3.2.1.4) it is reconstructible //.

Corollary (3.2.1.6)

A graph G in which the subgraphs G_i's in all the cards of the deck, have the same number of lines (q_i's are same, eventhough they may not all be isomorphic) is reconstructible.

3.3. Non-consecutive graphs

Definition (3.3.1)

Let G be a (p,q) graph, and its degree sequence be $d_1 \leq d_2 \leq \dots\dots \leq d_p$. Then the graph G is said to be a Non-consecutive graph if $d_{i+1}-d_i \neq 1$. (for all $i = 1,2,\dots,p-1$) see Figure (3.3.2).

Lemma (3.3.1)

Non-consecutive graphs are recognizable.

Proof

With the help of all G_i's we can find the degree sequence of G and if $d_{i+1}-d_i \neq 1$ (for all $i = 1,2,\dots,p-1$), then and only then G is a Non-consecutive graph //.

Theorem (3.3.1)

Non-consecutive graphs are reconstructible.

Proof

Let G be a Non-consecutive graph. We can find the degree sequence of the graph G with the help of the cards of deck. Let it be $d_1 \leq d_2 \leq \dots\dots \leq d_p$. Then as G is a Non-consecutive graph by the definition (3.3.1) $d_{i+1}-d_i \neq 1$ (for all $i = 1,2,\dots, p-1$). Choose any card from the deck. Let the card be G_i having q_i lines. Then $d_i = q-q_i$, where d_i is the degree of the point v_i. Then the degree sequence

of G_i will be given (not necessarily in the ascending order)

$$d_1 - \epsilon_{1i},\ d_2-\epsilon_{2i},\ \ldots .d_{i-1}-\epsilon_{(i-1)i},\ d_{i+1}-\epsilon_{(i+1)i}, \ldots\ d_p-\epsilon_{pi}.$$

$$\text{where}\quad \epsilon_{ji} = 1 \quad \text{if } v_jv_i \text{ is a line in G. for all } j = 1,2,\ldots,\hat{i},\ldots p.$$
$$= 0 \quad \text{otherwise.}$$

out of these $(p-1)$ constants ϵ_{ji}, for all $j = 1,2,\ldots,\hat{i},\ldots,p$. We have that exactly d_i must be 1 and $(p-1)-d_i$ must be zero as v_i is joined with exactly d_i points in G.

Let v_j be a point such that v_jv_i is a line in G for $j = 1,2,\ldots,\hat{i},\ldots,p$. Then $\epsilon_{ji} = 1$ and in G_i the degree of the point v_j will be $d_j - \epsilon_{ji} = d_j - 1$ but it can not be equal to the degree of any point in G. For, let, if possible, there exist a k such that $d_k = d_j - 1 \Rightarrow d_j - d_k = 1$, which is a contraduction and hence $d_j - \epsilon_{ji}$ is not the degree of any point in G. In order to reconstruct G we insert the point v_i in G_i. Then v_i must be joined to the point v_j's in G_i. In particular v_i must be joined to the point v_j in G_i having degree $d_j - \epsilon_{ji}$. But we have exactly d_i points as v_j in G_i which are having degrees not equal to the degree of any point of G. Hence we can uniquely join the point v_i to these d_i points and get the graph G and those points v_j which are not joined with v_i in G have $\epsilon_{ji} = 0$. They have degrees d_j in both G_i and G. Therefore there is no need to join them in the procedure for reconstruction. Hence we can uniquely reconstruct the Non-consecutive graph G //.

Corollary (3.3.1.1)

Euler and Odd graphs are reconstructible.

Proof

Euler and Odd graphs are also Non-consecutive graphs as $d_{i+1}-d_i$ = o or an even number. This implies that $d_{i+1}-d_i \neq 1$ (for all $i = 1,2,\ldots,p-1$).

Example (3.3.1)

A deck of cards of a graph G is given in Figure (3.3.1). With the help of these we reconstruct the graph G. See Figure (3.3.2).

Number of points in G is p = Number of cards = 8.

$$\text{Number of lines in G is } q = \frac{\sum_{i=1}^{8} q_i}{p-2} = \frac{12+7+10+7+10+12+10+10}{8-2}$$

$$= \frac{78}{6} = 13.$$

degree sequence $q-q_1, q-q_2, q-q_3, q-q_4, q-q_5, q-q_6, q-q_7, q-q_8$.

= 13-12, 13-7, 13-10, 13-7, 13-10, 13-12, 13-10, 13-10.

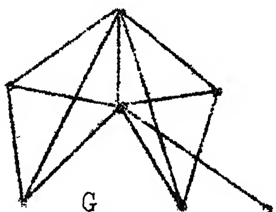
After rearranging in ascending order we have 1,1,3,3,3,3,6,6.

The degree sequence satisfies the condition $d_{i+1}-d_i \neq 1$ (for all $i = 1,2,\ldots 7$).

Hence by Definition (3.3.1) G is a Non-consecutive graph.

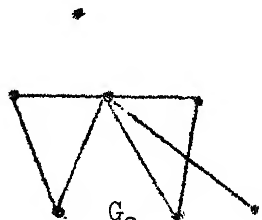
Now choose any card say G_5 and insert the point v_5. In order to get G we calculate the degree sequence of G_5, that is, 1,1,2,3,3,5,5.

Here exactly three points have degrees 2,5,5 not according to the degree sequence of G. Hence we have to join our point v_5 uniquely to these three points to get our graph G. Hence G gives the unique reconstruction. See Figure (3.3.2).



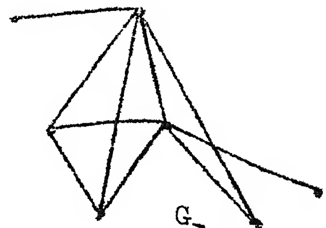
G_1

$q_1 = 12$



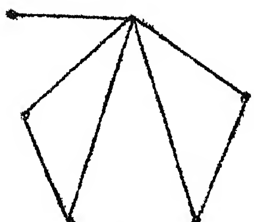
G_2

$q_2 = 7$



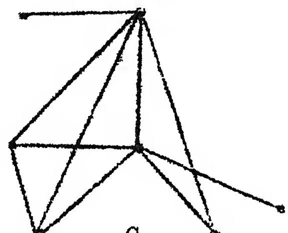
G_3

$q_3 = 10$



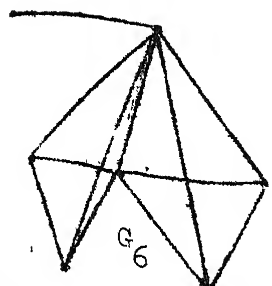
G_4

$q_4 = 7$



G_5

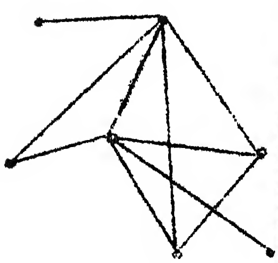
$q_5 = 10$



G_6

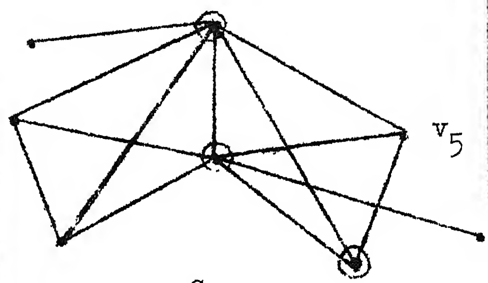
$q_6 = 12$

Figure (3.3.1)



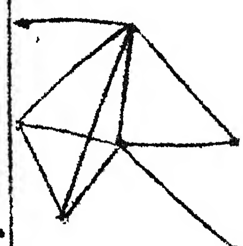
G_7

$q_7 = 10$



G

Figure (3.3.2)



G_8

$q_8 = 10$

In the next chapter we discuss the reconstruction of Complete bipartite graphs and list a number of unsolved problems.

Chapter 4

FURTHER DIRECTIONS

In this chapter we prove that Complete bipartite graphs are reconstructible. We list a number of unsolved problems. Some of them may be found in (14).

4.1. Complete bipartite graphs

Definition(4.1.1)

A bipartite graph G is a graph whose point set V can be partitioned into two subsets V_1 and V_2 such that every line of G joins a point of V_1 with a point of V_2.

Definition (4.1.2)

A bipartite graph G with partitions V_1 and V_2 of the point set V is said to be a Complete bipartite graph if every point of V_1 is adjacent to every point of V_2.

Let G be a Complete bipartite (p,q) graph, with partitions V_1 and V_2 of the point set V. Let V_1 and V_2 have p_1 and p_2 points respectively. Then we have

$$p = p_1 + p_2 \quad .....(4.1.1) \;, \quad q = p_1 \cdot p_2 \quad .....(4.1.2)$$

If we know p_1 and p_2, then we can uniquely determine the Complete bipartite graph G which is denoted by K_{p_1,p_2}. Lemma (4.1.1) given below shows that Complete bipartite graphs are recognizable.

Lemma (4.1.1)

G is K_{p_1,p_2} if and only if p_1 cards have K_{p_1-1,p_2} and p_2 cards have K_{p_1,p_2-1}.

Proof :

Let G be K_{p_1,p_2}. Then it has two partitions say V_1 and V_2 having p_1 and p_2 points respectively. If we remove any point from V_1, then we get a K_{p_1-1,p_2} subgraph and since V_1 has p_1 points, p_1 cards will have K_{p_1-1,p_2} and similarly p_2 cards will have K_{p_1,p_2-1}.

Conversely, let p_1 cards have K_{p_1-1,p_2} and p_2 cards have K_{p_1,p_2-1}. Then in all they are p_1+p_2 cards. Therefore $p = p_1+p_2$ and also p_1 cards have $q_i = (p_1-1)p_2 = p_1p_2-p_2$ and p_2 cards have $q_i = p_1(p_2-1) = p_1p_2-p_1$

Therefore :

$$\sum_{i=1}^{p=p_1+p_2} q_i = p_1(p_1p_2-p_2) + p_2(p_1p_2-p_1) = p_1p_2(p_1+p_2-2)$$

Therefore $$q = \frac{\sum_{i=1}^{p} q_i}{p-2} = \frac{p_1p_2(p_1+p_2-2)}{(p_1+p_2-2)} = p_1p_2$$

Now the degree sequence of G will be $q-q_i$, for all $i = 1,2,\ldots,p$.

$p_1p_2-(p_1p_2-p_2), \ldots\ldots p_1p_2-(p_1p_2-p_2)$;

$\ldots\ldots\ldots (p_1$ times$) \ldots\ldots\ldots\ldots\ldots$

$p_1p_2 - (p_1p_2-p_1), \ldots\ldots\ldots\ldots p_1p_2 - (p_1p_2-p_1)$

$\ldots\ldots\ldots (p_2$ times$) \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

or $\quad p_2, p_2, \ldots\ldots, p_2$; $\quad p_1, p_1, \ldots\ldots\ldots p_1$.

$\quad \ldots (p_1$ times$) \ldots \quad \ldots\ldots (p_2$ times$) \ldots\ldots$

Now take any subgraph K_{p_1-1, p_2}. Then its degree sequence must be $p_2, p_2 \ldots\ldots p_2$, $p_1-1, p_1-1, \ldots\ldots p_1-1$.

$\ldots (p_1-1$ times$) \ldots \quad \ldots (p_2$ times$) \ldots\ldots$

Now insert a point in it and then in order to get the degree sequence of G, we have to join it to all the points of degree p_1-1 in K_{p_1-1, p_2} which are p_2 in number. Then our G must be K_{p_1, p_2} //.

Theorem (4.1.1)

Complete bipartite graphs are reconstructible.

Proof :

Once we recognize the Complete bipartite graph, then it is easy to reconstruct. Knowing p_1 and p_2 the graph K_{p_1, p_2} is uniquely determined //.

4.2 Unsolved problems

Definition (4.2.1)

A spanning subgraph is a subgraph containing all the points of G.

Definition (4.2.2)

If a graph G has a spanning cycle Z, then G is called a Hamiltonian graph.

Problem (4.2.1)

Show that Hamiltonian graphs are recognizable.

Problem (4.2.2)

Show that Hamiltonian graphs are reconstructible.

Definition (4.2.3)

A cut point of a graph is one whose removal increases the number of components.

Definition (4.2.4)

A non-separable graph is connected nontrivial, and has no cut points.

Definition (4.2.5)

A block of a graph is a maximal nonseparable subgraph.

Problem (4.2.3)

Show that separable graphs with end points are reconstructible. (Even the reconstruction of separable graphs with two blocks, one of which is K_2, would be a great achievement.).

A solution to problem (4.2.3), combined with Bondy's reconstruction (11) of separable graphs without end points, would leave us with non-separable graphs to reconstruct.

Problem (4.2.4)

Let R be a recognizable class of graphs. Show that all graphs are reconstructible if R is reconstructible.

Definition (4.2.6)

A graph is said to be embedded in a surface S when it is drawn on S so that no two lines intersect.

Definition (4.2.7)

A graph is planar if it can be embedded in the plane.

Definition (4.2.8)

A maximal planar graph is one to which no line can be added without losing planarity.

Problem (4.2.5)

Show that maximal planar graphs are reconstructible.

Problem (4.2.6)

Show that planar graphs are reconstructible.

Problem (4.2.7)

Find the minimum number of subgraphs G_i which is necessary to reconstruct a tree.

The above problems remain open when we turn to edge reconstruction. Indeed, since a graph without isolated points is edge reconstructible

if it is reconstructible, it is advisable to attempt the edge version of those problems first. Rather th n state the edge analogus of all the previous problems, we pick out three of particular interest.

Problem (4.2.8)

Show that bipartite graphs are edge reconstructible.

Problem (4.2.9)

Show that planar graphs are edge reconstructible.

Problem (4.2.10)

Show that Hamiltonian graphs are edge reconstructible.

The Lavasz and Muller theorems were major advances, but they differ from most other reconstruction results in that their proofs are existential in nature. Constructive proofs might well be enlightening.

Problem (4.2.11)

Give constructive proofs of the Lovasz and Muller theorems.

Although the digraph, reconstruction conjecture has been disproved, we could of course, ask about the reconstructibility of various classes of digraphs. But we refrain, and instead state two problems of Stockmeyer (126) concerning his counter examples.

Problem (4.2.12)

Find the odd-ordered non-reconstructible tournaments other then the Stockmeyer's examples.

Problem (4.2.13)

Are all even-ordered counter example pairs of tournaments complements of one another ?

Definition (4.2.9)

The Eulericity E(G) of a graph G is the smallest number of Eulerian subgraphs whose union is G.

Problem (4.2.14)

Show that the Eulericity of a graph G is reconstructible.

Problem (4.2.15)

Show that bipartite graphs are recognizable.

Problem (4.2.16)

Show that bipartite graphs are reconstructible.

Definition (4.2.10)

A k-partite graph G is a graph whose point set V can be partitioned into k-subsets $V_1, V_2, \ldots, V_k$ such that every line of G joins V_i with V_j, (for all $i, j = 1, 2, \ldots k$ and $i \neq j$).

Problem (4.2.17)

Show that k-partite graphs are recognizable.

Problem (4.2.18)

Show that k-partite graphs are reconstructible.

Definition (4.2.11)

A graph G is a k-degreed graph if its degree sequence has only k, distinct integers.

Example (4.2.1)

K_4-(e) which has the degree sequence (2,2,3,3) is a bidegreed graph.

Problem (4.2.19)

Show that bi-degreed graphs are reconstructible. (This is trivial unless the two degrees differ by one).

Problem (4.2.20)

Show that tri-degreed graphs are reconstructible.

Lastly, we hope that Reconstruction-Conjecture will be settled down by the following problem.

Problem (4.2.21)

With the help of results of problems (4.2.19) and (4.2.20) by mathematical induction show that all finite, simple, undirected, k-degreed graphs on three or more points are reconstructible.

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